

# Concerns on Monotonic Imbalance Bounding Matching Methods<sup>1</sup>

by

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## Summary

*Concerns are expressed for the Monotonic Imbalance Bounding (MIB) property (Iacus et al. 2011) and for MIB matching because i) the definition of the MIB property leads to inconsistencies and the nature of the imbalance measure is not clearly defined, ii) MIB property does not generalize Equal Percent Bias Reducing (EPBR) property, iii) MIB matching does not provide statistical information available with EPBR matching.*

Imbalance Bounding (IB) matching is examined but the findings and the comments remain valid for MIB matching. Familiarity of the readers with Iacus, King and Porro (2011, hereafter IKP 2011) and the notation therein is assumed.

*On the definition of the IB property*

We use the IB-Definition obtained from the authors in a recent communication.

**IB-Definition:** Let  $f$  be any measurable function and  $D(\cdot, \cdot)$  any *measure of imbalance* that can be bounded by a scalar. Assume (a) fixed sizes of the random samples  $n_T$ ,  $n_C$ , (b) fixed distributions of  $\mathbf{X}$ ,  $P_T$  for the treated population and  $P_C$  for the control population, (c) a fixed matching method is used. If for a given value of  $\delta$  we obtain matched samples of sizes  $m_T$  and  $m_C$  such that

$$D[f(\mathcal{X}_{m_T}), f(\mathcal{X}_{m_C})] \leq \delta, \quad (1)$$

then we have the property IB;  $\mathcal{X}_{m_T}$ ,  $\mathcal{X}_{m_C}$  are, respectively, the matched-treated and matched-control data.

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<sup>1</sup>Appeared as online supplement to the *JASA* paper by Iacus, King and Porro (2011) with response from the authors.

Since IB property (1) is dependent on a  $\delta$ -value determined in advance (ex-ante), the following situations will occur:

*a)* for fixed treatment and control populations and a statistician with ex-ante  $\delta = \delta_1$  the matching method has not the IB property, but for another statistician with ex-ante  $\delta = \delta_2 > \delta_1$  and  $\delta_2$  large enough the same matching method has the IB property. Consequently, the two statisticians will be in disagreement on whether the matching method has the IB property or not, leading to an inconsistency. Thus, the class of IB matching methods is not well defined.

*b)* For readers inclined to justify the inconsistency in *a)* from the subjective choice of different  $\delta$ -values by the two statisticians, consider a fixed matching method, one statistician and two sets of treatment and control populations with distributions  $P_{T_i}$  and  $P_{C_i}$  and bounds  $\delta_i$ ,  $i = 1, 2$ . This statistician may find that the matching method satisfies IB property (1) for the distributions and the bound  $(P_{T_1}, P_{C_1}, \delta_1)$  but not for the distributions and the bound  $(P_{T_2}, P_{C_2}, \delta_2)$ . Does the matching method have the IB property in this situation? This will hold if the IB definition is population dependent but it is not the case since, according to the authors, no assumptions are needed on the populations' distributions for IB property to hold (IKP 2011, p. 346, section 2.2, lines 2-4).

Looking at (1) any graduate student in statistics would ask “What is the probability that (1) holds?” given that  $f$  is a measurable function not necessarily constant. If this probability is equal to 1, questions will arise concerning the applicability of the method for all populations' distributions, suggesting that IB-definition is population dependent. If this probability is less than 1, IB definition is data dependent and therefore, for a fixed  $\delta$ -value, the matching method may have the IB property for one data set but this may not hold for a different data set from the same population.

Note that in IB definition (IKP 2011, p. 347),  $D(x, y)$  denotes a distance between  $x$  and  $y$  but in the examples following this definition  $D(x, y) = E(x - y)$  and  $D(x, y) = |x|$  are not distances;  $E$  denotes expected value. In IB definition (1) “D is any measure of imbalance” but no precise definition of what this means is available. There are no guidelines for the choice of the  $\delta$ -value and a natural approach for its selection presented below makes IB definition data dependent.

*IB and EPBR matchings-Does IB matching generalize EPBR matching?*

Our main argument against the claim in IKP (2011) that IB property generalizes Rubin’s EPBR property (Rubin 1976) is that IB loses the EPBR property of affine invariance with respect to linear combination of means. In a recent communication the authors provided the arguments that follow, in order to show that IB property is “a mathematical generalization of EPBR property.” Their motivation for the IB definition is EPBR definition

$$\mu_{T^*} - \mu_{C^*} = \gamma(\mu_T - \mu_C), \quad 0 < \gamma < 1, \quad (2)$$

i.e. the expected value of the difference of matched samples means,  $\mu_{T^*} - \mu_{C^*}$ , is a proportion  $\gamma$  of the expected value of the difference of random samples means,  $\mu_T - \mu_C$ . For elements  $x, y$  let  $D(x, y) = x - y$  and for a random vector  $A$  set  $f(A) = E(A)$ ;  $E(A)$  denotes the expected value of  $A$ . Then, EPBR property (2) is rewritten in the IB-like notation

$$D[f(\mathcal{X}_{m_T}), f(\mathcal{X}_{m_C})] = \delta, \quad (3)$$

with

$$D[f(\mathcal{X}_{m_T}), f(\mathcal{X}_{m_C})] = \mu_{T^*} - \mu_{C^*}, \quad \delta = \gamma(\mu_T - \mu_C); \quad (4)$$

$\mathcal{X}_{m_T}, \mathcal{X}_{m_C}$  denote matched sub-samples and  $f(\mathcal{X}_{m_T}), f(\mathcal{X}_{m_C})$  denote the expectations of the matched sample means. Finally, the equality sign in (3) is replaced by “ $\leq$ ” and the authors’ conclusion is that “In this way, we have shown that IB is a direct mathematical generalization of EPBR.” However,  $E(A)$  is a functional of the cumulative distribution of  $A$  and this also holds for the expectations’ differences in (2) *but* is not reflected in (3) and the IB definition (1) which only involve measurable functions of the data. Thus, the authors’ arguments do not show that IB property is mathematical generalization of EPBR property.

Irrespective of the last sentence, using the authors’ motivation we examine whether statistical information other than affine invariance is lost with IB matching methods. Going one step further from  $\delta$ ’s definition in (4) we obtain from the EPBR property

$$\delta = \gamma(\mu_T - \mu_C) = \gamma D[f(\mathcal{X}_{n_T}), f(\mathcal{X}_{n_C})], \quad (5)$$

$\mathcal{X}_{n_T}, \mathcal{X}_{n_C}$  are random samples. It may occur, for example, that a practitioner uses an IB matching method with (small)  $\delta$ -value  $10^{-4}$ , but the value of  $D[f(\mathcal{X}_{n_T}), f(\mathcal{X}_{n_C})]$  is  $10^{-6}$ . Equation (5) suggests using  $D[f(\mathcal{X}_{n_T}), f(\mathcal{X}_{n_C})]$

to determine an appropriate  $\delta$ -value but this will introduce random sample dependence in the IB definition unless  $f$  is constant.

Unlike the IB property, EPBR property (2) provides via  $\gamma$  information on the improvement of expected matched means' difference compared to the expected random means' difference. In the IB definition a subjective  $\delta$ -value is used, it is not clear in IKP (2011) what this value should be and there is no comparison with the  $D$ -value obtained via  $f$  for random samples. Moreover, with the EPBR property both sides in (2) have the same sign. This information is also lost with a distance  $D$  in the IB definition. Thus, there is loss of statistical information when using IB matching methods instead of EPBR matching methods.

EPBR property (2) is clearly moments' property and mild moment conditions are provided in Yatracos (2013) for EPBR property to hold for a class of matching methods, thus relaxing the criticism that EPBR holds only under restricted distributional assumptions (IKP 2011, p. 346).

The concerns presented for IB matching methods hold also for MIB matching methods (IKP 2011, p. 347) for which (1) holds with data  $\mathcal{X}_{m_T(\pi)}$ ,  $\mathcal{X}_{m_C(\pi)}$  and upper bound  $\gamma_{f,D}(\pi)$  (instead of  $\delta$ ) depending on a tuning parameter  $\pi$ ;  $\gamma_{f,D}(\cdot)$  is monotonically increasing in  $\pi$ .

## References

- Iacus, S. M., King, G. and Porro, G. (2011) Multivariate Matching Methods that are Monotonic Imbalance Bounding (MIB) *JASA*, **106**, p. 345-361.
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